Ch. 1: Motivating Examples

Recall: $\frac{dy}{dx} = rate$

$$\frac{dT}{dt} = -0.5(T-70) \quad \frac{°F}{\text{minute}}$$

Newton's Law of cooling

Rates come up everywhere!

a) *Populations, Savings Accounts, Newton's Law of cooling, Mixing Problems, melting ice (see handout)*

$$\frac{dP}{dt} = 0.03P \quad \frac{\text{people}}{\text{yr}}$$

3% relative growth rate

$$\frac{dA}{dt} = 0.02A \quad \frac{\text{dollars}}{\text{yr}}$$

2% interest rate (compounded continuously)

$$\frac{dA}{dt} = 0.02A + 100 \quad \frac{\text{dollars}}{\text{yr}}$$

2% interest rate plus 100 deposited per year

b) *Free-fall (no air resistance):*

$$mv' = F_g = -mg$$

*Initial **V**alue **P**roblem* (IVP)

$$v' = -g$$

$$v(0) = 0$$ ← DROPPED

Newton's Law

Force = $ma = mv' = m\dot{v} = m\frac{dv}{dt}$

↑ ↑ ↑
ASIDE: 3 ways you'll see this

Force due to gravity = mg (downward)

So

ma = FORCE = $-mg$

$m\frac{dv}{dt} = -mg$

$$\boxed{\frac{dv}{dt} = -9.8}$$

$$\Rightarrow \boxed{v(t) = -9.8t + C}$$

c) ...with air resistance

$$mv' = F_g + F_A = -mg - rv$$

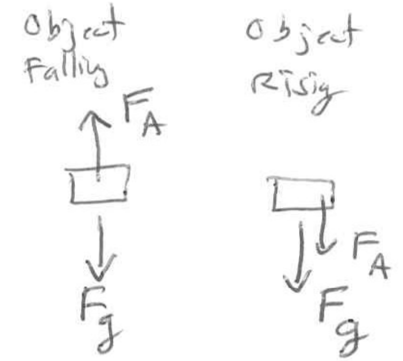
$$v' = -g - \frac{r}{m}v$$

$$v(0) = 0$$

ONE MODEL
↳ {"Air resistance proportional to velocity"

r = proportionality constant

$\frac{dv}{dt} = -g - \frac{r}{m}v$ CAN'T INTEGRATE!!

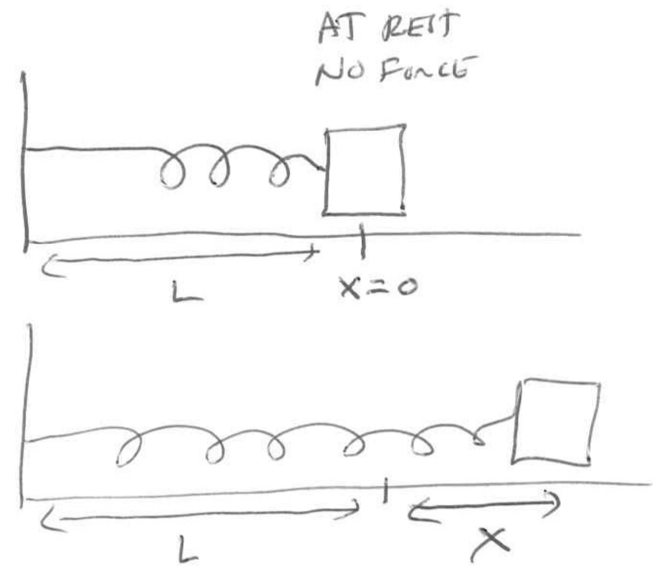


d) Mass-Spring Example:

$$Force = -kx$$

$$m\,x'' = -kx$$

k = force to hold mass "one unit" away from natural length



It turns out that one solution to this is $x(t) = cos(\omega t)$

$$x' = -\omega \sin(\omega t)$$

$$x'' = -\omega^2 \cos(\omega t)$$

$$-m\omega^2 \cos(\omega t) \overset{?}{=} -k\cos(\omega t)$$

↑ NEED TO BE SAME! ↑

$$m\omega^2 = k$$

$$\Rightarrow \omega^2 = \frac{k}{m}$$

$$\omega = \pm\sqrt{\frac{k}{m}}$$

$$x(t) = \cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

SIMPLE HARMONIC OSCILLATOR

$\sqrt{\frac{k}{m}}$ = natural frequency $\frac{\text{radians}}{\text{sec}}$

$\frac{2\pi}{\omega}$ = wavelength

e) *Circuits (read lecture notes,* ← FROM MY WEBSITE

follow instructions on HW):

$$V = Rq' + \frac{1}{c}q + Lq''$$

IN HW PROBLEM, $V_L = 0$ $(L=0) \Rightarrow$ $V = R\frac{dq}{dt} + \frac{1}{c}q$

part (a) → $\frac{dq}{dt} = ??$

part (b) $V_c(t) = \frac{q(t)}{c}$ $\Rightarrow$ $\frac{dV_c}{dt} = \frac{1}{c}\frac{dq}{dt}$

USE THIS

Slope/Direction Fields

Recall: $\frac{dy}{dx} = slope$

We can visualize slope!

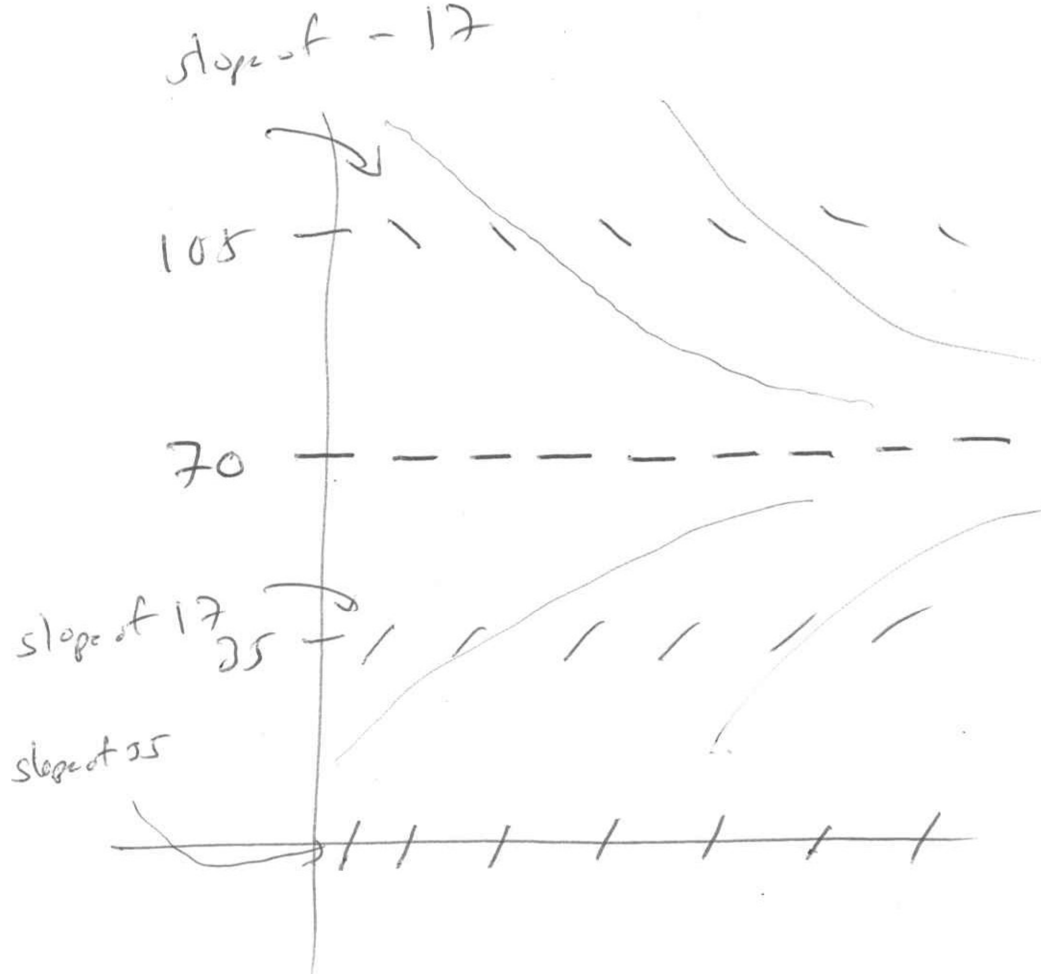
Example 1:

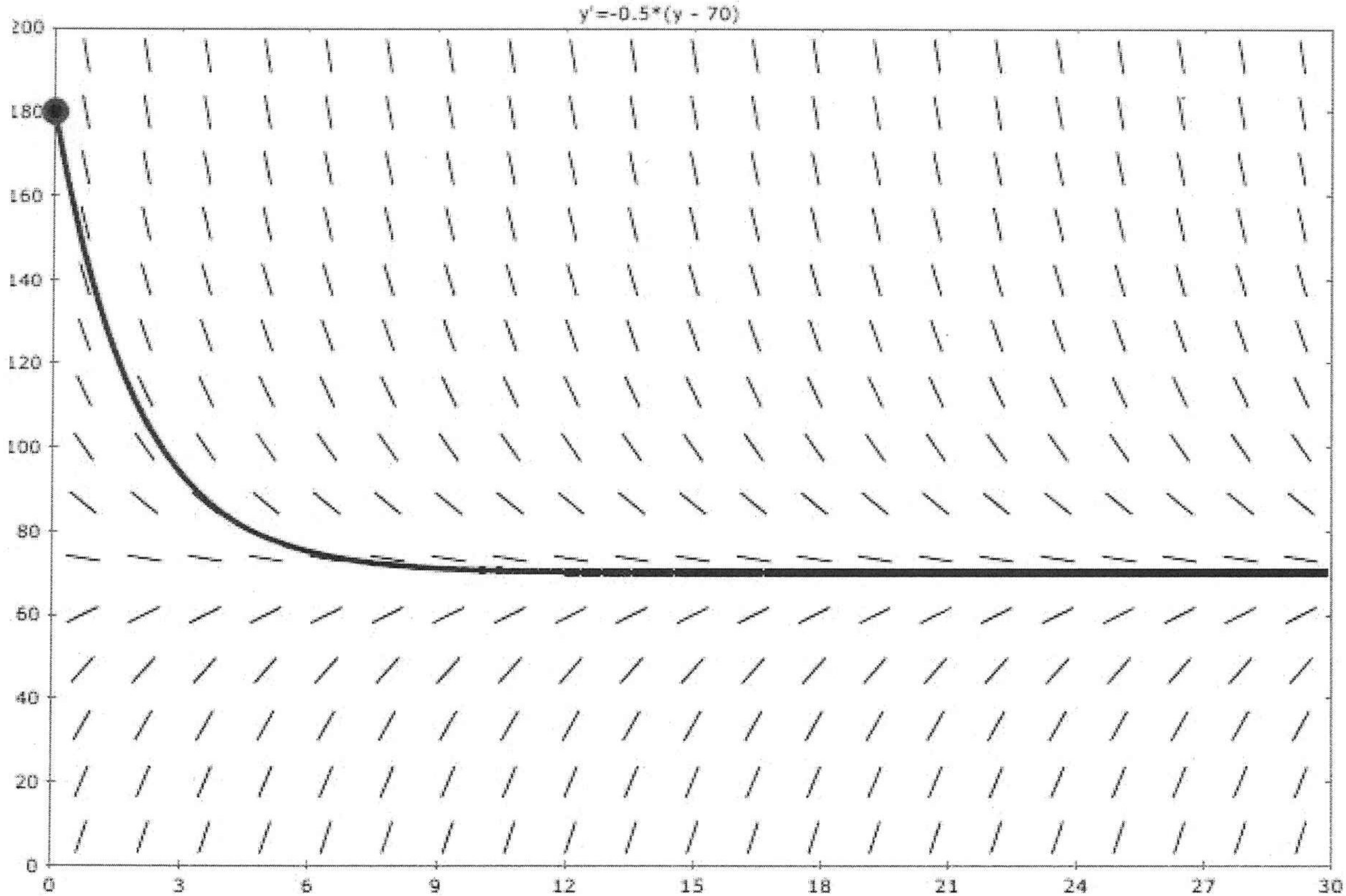
$$\frac{dT}{dt} = 0.5(70 - T)$$

	T=0	T=35	T=70	T=105
t=0	35	17	0	−17
t=10	35	17	0	−17
t=20	35	17	0	−17
t=30	35	17	0	−17

↑

autonomous (doesn't depend on t)





y'=-0.5*(y - 70)
200
180
160
140
120
100
80
60
40
20
0
0
3
6
9
12
15
18
21
24
27
30

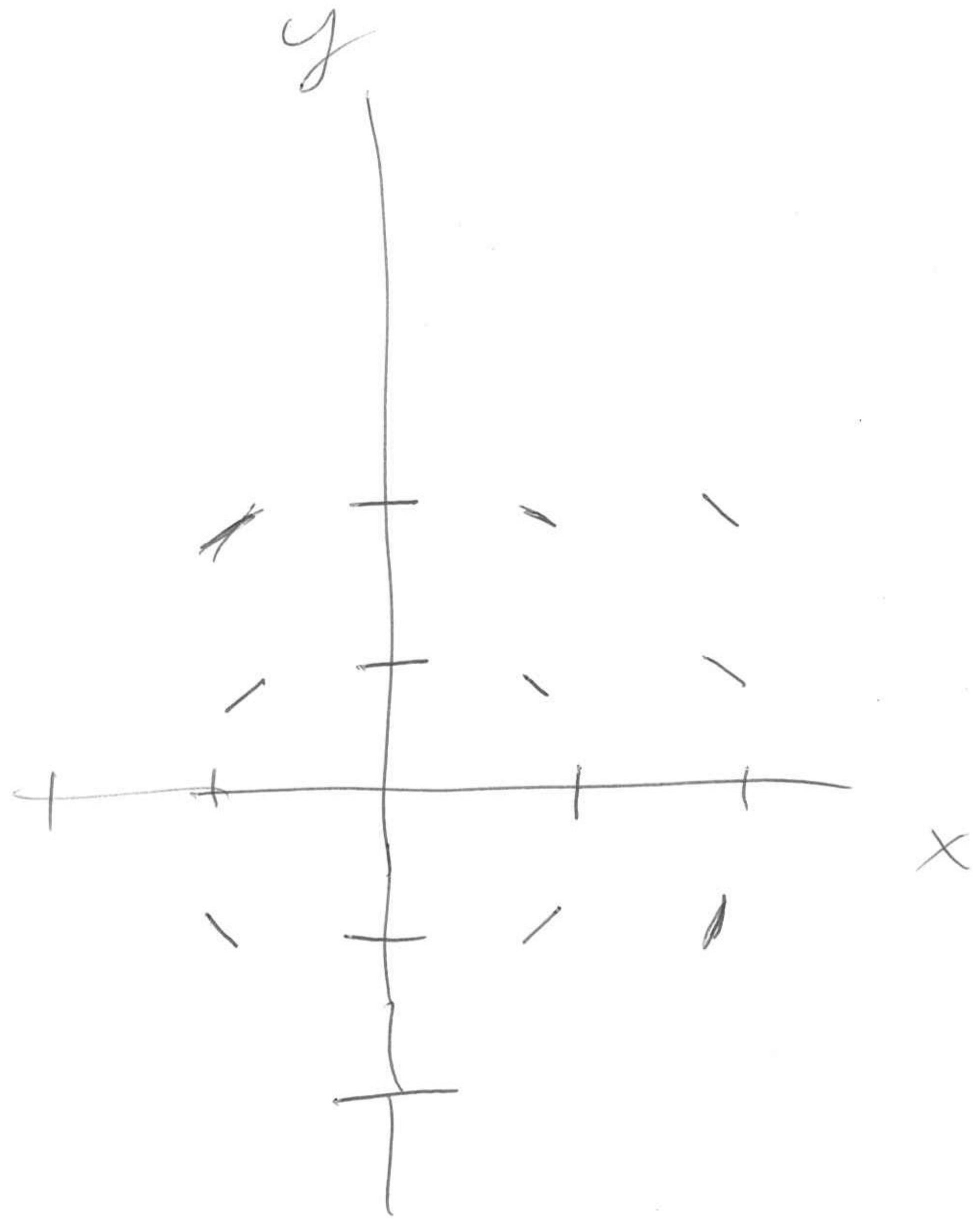
Example 2:

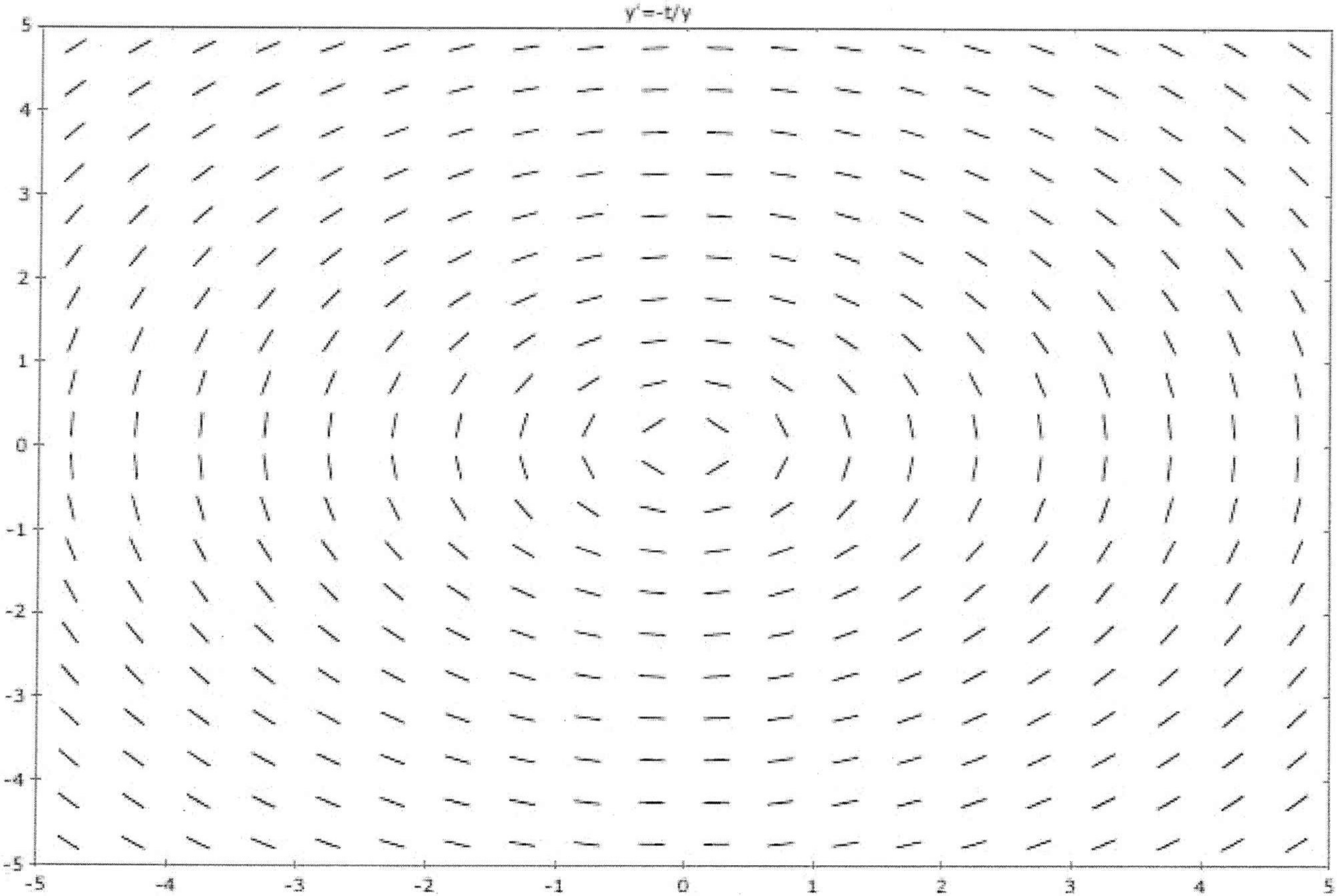
depends on both x & y

$$\frac{dy}{dx} = -\frac{x}{y}$$

	y=-1	y=0	y=1	y=2
x=-1	−1	?	1	½
x=0	0	0	0	0
x=1	+1	?	−1	−½
x=2	2	?	−2	−1

vertical tangent




y'=-t/y
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
0
1
2
3
4
5